DQP Assignment Library: Mathematics

Calculus I

Assignment Title:

Keychain Ziplines: A Practical Way to Study Velocity in the Calculus Classroom

Activity by Lisa Driskell and Audrey Malagon

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DQP Proficiencies:

This assignment is intended as a formative assessment in the areas of

Applied Learning

Context of the Assignment:

This activity was designed as a first day activity for college calculus courses and tested in a variety of college calculus classrooms. College calculus courses are typically the introductory course to a mathematics or computer science major, but also serve many students in the natural sciences. Most students are in their first year of college, but there are a few upperclass students as well.

Learning Objectives:

- Review calculations of average velocity
- Introduce concept of instantaneous velocity
- Have students explore the connections between average and instantaneous velocity
- Create a framework for the discussion of limits

Assignment Details, Reflections, and Outcomes:

The details of the activity as well as reflections and outcomes are published in the article "Keychain Ziplines: A Practical Way to Study Velocity in the Calculus Classroom" by Lisa Driskell and Audrey Malagon in PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies, 23:7, 590-597. The article is reprinted here by permission of Taylor & Francis (http://www.tandfonline.com)

The article is followed by an instructor and student guide to the activity.

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Keychain Ziplines: A Practical Way to Study Velocity in the Calculus Classroom

Lisa Driskell and Audrey Malagon

Abstract: Using materials that are easy to procure, students create a zipline for a weighted keychain. They measure distances and times to calculate average velocities and explore the idea of instantaneous velocity. This inquiry-based calculus activity has been tested as a first day activity in classrooms at a variety of institutions and has received positive student and instructor feedback.

Keywords: Zipline, calculus, velocity, tactile learning, inquiry-based learning, average velocity, instantaneous velocity.

1. INTRODUCTION

Perhaps no topic lends itself better to a hands-on demonstration than the idea of average and instantaneous velocity. The trouble, however, is that common examples such as objects in free fall or balls rolling down ramps are hard to reproduce in the classroom in ways that allow for accurate student measurement. Our keychain zipline activity provides an easy-to-build example of a moving object for which students can accurately record time, distance, and velocity measurements (Figure 1). It introduces students to the idea of instantaneous velocity and sets up a framework for the discussion of limits. When used as a first day activity in Calculus I, it allows students to meet each other and sets the tone for an active class. This activity has been tested in classes ranging from 15 to 35 students and in public university and private liberal arts college settings.

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Figure 1. A keychain moves down the zipline as students work together to take time measurements using a stopwatch on a phone (color figure available online).

2. OBJECTIVES

The objectives for this activity are twofold: learning objectives in calculus and goals for classroom dynamics. Regarding calculus, we want to review average velocity calculations, introduce the concept of instantaneous velocity, and have students explore the relationship between the two as we set up a framework for the study of limits later in the course. We also wish to foster students' creativity by introducing an inquiry-based learning (IBL) activity. The zipline has them thinking and interacting in the classroom on the first day of class.

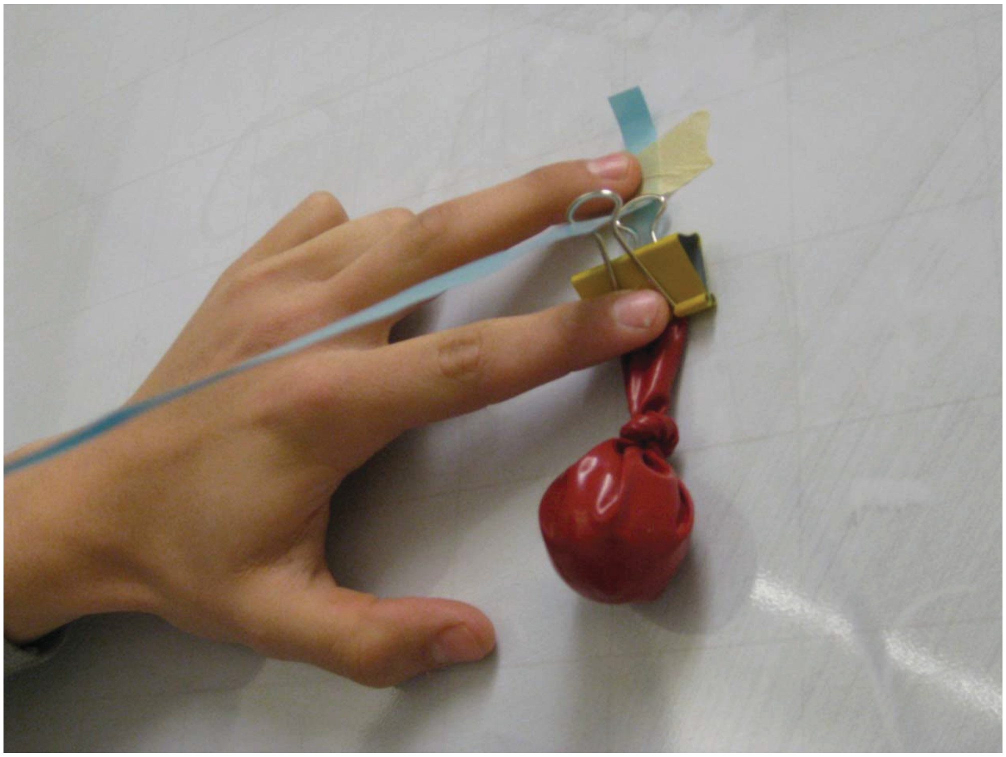


Figure 2. A zipliner (binder clip with a weight) is in position to be released on the zipline (smooth gift wrapping ribbon) (color figure available online).

3. SET-UP AND MATERIALS

To set up the activity, we divide the class into groups of three or four and give each group the following supplies (which can be found at local dollar or discount stores).

1. Zipliner: heavy keychain or one with a weighted object such as golf ball (duct tape works great!), a balloon with a weighted ball inside attached to a binder clip (Figure 2), a small padlock, or similar object.
2. Tape measure.
3. Length of smooth gift wrapping ribbon (at least 6–8 feet or let students cut their own from a whole spool).
4. Scissors.
5. Tape.
6. Marker that writes on the ribbon.
7. Stopwatch (smart phone apps work fine here).

Students should first build and test the zipline. The ribbon can be attached to walls, desks, chairs, or the floor, but should be taut and should be angled so the zipliner takes at least 2 seconds to travel the entire length. Building and testing should take less than 5 minutes.

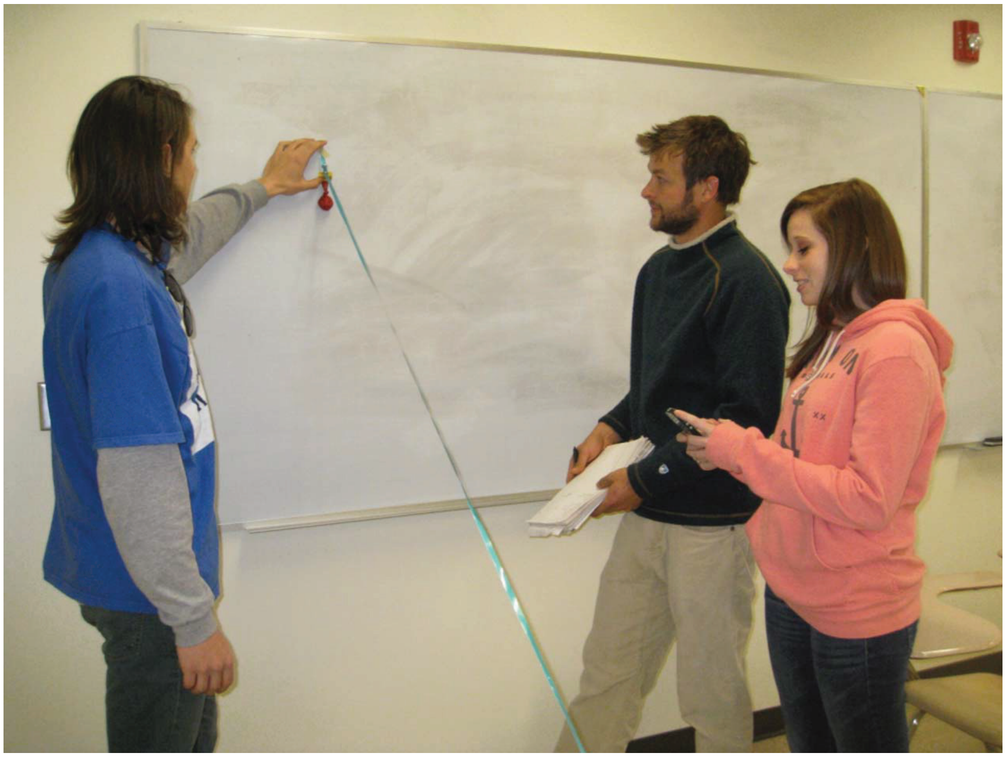


Figure 3. Students record data and collaborate to determine the average velocity of the zipliner (color figure available online).

4. THE ACTIVITY

When the ziplines are built, we ask the students what information they are able to determine with just the materials provided. Ambitious groups calculate angles and slopes, but almost all record time and distance. We keep this exploration part to no more than 5 minutes and then lead into a large class discussion and review of average velocity.

Following this review, students calculate and record the average velocity of the ziplines for the entire trip. (In larger classes, we provide a handout for them to record this information.) This is done by having the students use a stopwatch or stopwatch app to time the zipliner as it travels. In our classes, we have had students start and stop the time when they see the zipliner start and stop, but with advanced smartphones, students could also create movies and use the frame speed to get more accurate results.

We then instruct the students to mark the point on the ribbon one-third of the way from the bottom of the zipline. The students record the average velocity for the top two-thirds of the trip as well as the bottom one-third of the trip and compare these values along with their initial recording of average velocity (Figure 3). Students are often amazed that these values are different! Many have viewed quantities that they can calculate as constant. The idea that velocity can change is eye opening for them. Since calculus is the study of change, this is an important leap! It was not until we used the zipline activity that we saw this pitfall in student reasoning, but now that we have, we better understand some of the struggles our students have with concepts in calculus. We have found our students much more interested in calculating velocity at a particular moment once they realize it may in fact vary.

Once students agree that velocity changes, we begin to talk about instantaneous velocity. We loosely define this as a large group and give an illustrative example. One of our favorite examples comes from asking a commuting student how far she drove to class and how long it took her. We calculate her average velocity and then ask if she traveled exactly that speed the entire time. "Of course not!" she replies. So when we ask how fast she was going at a specific time, the students begin to understand that this is not a trivial question. They are ready to investigate instantaneous velocity.

We present them now with the big question, "Using only the tools you have, can you determine how fast the zipliner is traveling the moment it passes the two-third mark on your ribbon?" The first time we used this activity we asked them to find the instantaneous velocity, but students were not ready to take this step. The beauty of phrasing the question as, "Is it possible . . .," is that both yes and no answers can steer the class towards a discussion of limits. We are careful not to answer this question just yet, but instead leave the students to investigate for the remainder of the class time. Some students argue that it is not possible because they will always need an interval, and others argue that they can get close enough by looking at a much smaller interval. Near the end of class, we ask them to write a "blog" for the next class summarizing the activity and discussing "the big question." (We have not had the students post their blogs online, but we have had much more success phrasing this as a blog-style writing assignment than asking for written summaries of the activity and questions.) The entire activity can be completed in a 50 minute class period with a discussion the following class period.

5. STUDENT HANDOUT

Part I. Construction

Use the materials in your kit to construct a keychain zipline out of ribbon. Run a few test runs and then secure the ends with tape. The keychain should glide quickly and easily down the zipline, but not too fast! (You will be timing intervals later.)

Part II. Average Velocity

Record the length of your zipline. Be sure to use appropriate units!

Length: ______________________________

Record how long it takes the keychain to travel the entire zipline. Use seconds (and fractions of seconds) for your units.

Time: ______________________________

The average velocity of the keychain during this trip can be found using the formula

$$\text{average velocity} = \frac{\text{distance traveled}}{\text{time to travel}}.$$

Find the average velocity. What are the units?
Average Velocity: ______________________________

Part III. Which end is faster? Find the point on the ribbon that is 1/3 of the distance from the bottom of the zipline. Mark this with your marker on the ribbon. You now will find the average velocity for the keychain on the top 2/3 of its trip and on the bottom 1/3. What will you measure in order to do this? Record and explain your calculations below.
Average Velocity for top 2/3: ______________________________
Average Velocity for bottom 1/3: ______________________________
Were these the same? How do they compare to each other and to the average velocity of the total trip you found in Part II?

Part IV. Instantaneous Velocity
We have been able to measure average velocities by calculating distance traveled and the time it took to travel that distance. Is it possible to know exactly how fast the keychain is traveling the moment it passes the 2/3 mark? If so, find this instantaneous velocity. If not, explain why not.

Writing Assignment Prepare a blog entry discussing today's activity. Summarize your group's findings and discuss the question in Part IV. Please type, print and bring your blog to the next class.

6. OUTCOMES AND STUDENT RESPONSES

In their written responses, students are consistently able to summarize the activity and review average velocity correctly. Their responses show that they are pondering the idea of instantaneous velocity as a limit but they are often not able to articulate this until the follow up discussion. In the future, we plan to give students more opportunity to investigate by asking them to calculate the average velocities on multiple intervals around the two-third mark or by asking them to calculate the instantaneous velocity just before the end of the string. Below are selected student responses:

- We have to compute an average velocity over an incredibly small time interval that is not accurately measurable with our class setup.
- . . . it is actually impossible to calculate instantaneous velocity, because... you need intervals for average velocity but we are not trying to get average around a certain point and you can get closer and closer and closer to the right answer by narrowing down the intervals.
- Anything we came up with was always an average which is not instantaneous.

Not all students gave insightful answers, of course, but we have found this activity engages almost all students in thinking more seriously about the question of instantaneous velocity, especially since it is not immediately answered, as it would be in a lecture. We have also found that using a tactile activity gives students permission to explore and think more freely since they do not have the preconceived notion that there is a right answer.

In addition to increasing students' understanding of instantaneous velocity, we have found that using the zipline activity on the first day of class has increased student participation throughout the semester. Students were more open to inquiry-based activities in other units (though we did not teach a strictly IBL calculus course) and more thoughtful in other investigative writing assignments. The student response has been overwhelmingly positive, with some even mentioning the activity in end-of-course evaluations. We've selected a few responses from recent blog assignments below:

- . . . you're forced to think on the first day of college.
- I woke up thinking 'I have calculus. Drat.'Instead, I found myself, not doing problems already written on a newly cleaned chalkboard, but rather, an activity which in real life, could be used to make one incredible ride. Long live the zip line!
- Today was actually a lot of fun and I am excited once again to be in a math class!!!
- If that's what the first class was like, I'm excited for the rest of the semester!

7. FOLLOW UP ACTIVITIES

There are many options to follow up this activity. We have found that a class discussion on instantaneous velocity as the limit of average velocities is well received the class after this activity. We might ask students to share their individual blog responses and build on those or open up the discussion again for a few minutes. We have found that it is essential in these discussions to never use the word velocity alone. Our students can now differentiate between average and instantaneous and we support this by using those adjectives to clarify whichever we are discussing. We have also graphed keychain position versus time and discussed graphical representations of average and instantaneous velocity as secant and tangent line slopes. The zipline activity gives the students a much better framework for these discussions, and we see improved conceptual understanding of limits, velocity, and derivatives.

An alternate extension activity, which we have not yet tested, is to build a giant zipline as a class that would take at least 15 seconds to travel. This would allow the class to accurately measure even more intervals than on their individual ziplines, but of course requires more set up and materials.

As the semester progressed, we often cited the activity when introducing new topics, which provided a visual reference for the students. For example, we related the notion of Δx approaching zero in the limit of a Riemann sum to the parallel procedures in the zipline activity. In addition, we referred to the distance traveled by the zipliner when determining net change. Having the zipline activity as a reference helped students connect different concepts throughout calculus.

8. CONCLUSIONS

The keychain zipline has been a fun and instructive way to start a Calculus I course, and we have found it gives students an increased understanding of instantaneous velocity. In order to do this activity right away, we have redesigned our syllabi to skip the traditional review of functions, incorporating the review later as needed. Students are excited about studying calculus, comfortable in an interactive environment, and curious about future topics.

ACKNOWLEDGEMENT

We would like to thank our engineering consultant, Mark Malagon, for his extensive and practical knowledge of gift wrapping ribbon strength and friction properties.

BIOGRAPHICAL SKETCHES

Lisa Driskell is an Assistant Professor of Mathematics at Colorado Mesa University. She teaches calculus and several other math courses. She is the advisor to the CMU Math Club, which plans Math Extravaganza, a day of hands-on math activities for high school students. She received her Ph.D. from Purdue University in 2010.

Audrey Malagon received her Ph.D. from Emory University in 2009 and is an Assistant Professor of Mathematics at Virginia Wesleyan College. She teaches a variety of math courses including calculus and incorporates IBL techniques whenever possible. She also advises the student math club and hosts Monday Night Math, a collaborative study time for students and faculty.

Lisa and Audrey have been exploring mathematics together since they were roommates at the Grand Valley State University REU program in 2002. They are both Project NExT fellows.

Keychain Ziplines

Lisa Driskell, Colorado Mesa University
Audrey Malagon, Virginia Wesleyan College

Instructor's Guide

Time: Approximately one 50-minute class period.

Materials: For each group of 3-4 students, prepare a bag with the following

- Heavy Keychain (weight with a golf ball for example), Padlock, Balloon filled with bouncy ball and clip, etc.
- 5-6 feet of ribbon or string, wide enough to mark on
- Markers
- Stopwatch (or have students download a stopwatch app prior to class)
- Tape measure
- Masking Tape
- Copy of Student Handout

Instruct students to construct a zipline for their keychain, and do a few test runs. The keychain should move quickly and easily down the ribbon, but not so fast that they will have trouble timing later on. (Minimum 1-2 seconds) Once they are happy with the zipline and you have checked their time, have them secure the ends to floor/desk/wall with tape.

Have them time how long it takes to travel the zipline. Then ask what information they can determine with this and their tape measure. (Lead to average velocity). Allow for small group discussion and then large group discussion. Check that they understand average velocity and are using appropriate units.

Pass out handout

Have students measure the length of their zipline and record on student worksheet. Have them time how long it takes for the keychain to travel the entire distance and then find the velocity.

Next they will find the location 1/3 of the way from the bottom and mark this and find how fast it travels in the first 2/3 of the journey and the last 1/3. Check their work and help as needed. Ask them about the differences in the two velocities and why they are different. Students who have had physics may be able to provide some insight into why they get these results. You can also discuss that velocity starts at 0 and ends at something positive.

Return to whole class discussion and loosely define instantaneous velocity together. Use an example such as a student's drive to class to distinguish between instantaneous and average velocity. (How long did it take you to get here? How far? What was your average velocity then? But did you drive exactly that speed the entire time?)

Finally, they will be asked to find the instantaneous velocity at the 1/3 point they have marked. Alternately you can ask them about the velocity at the very end of the zipline before it hits the wall. Let students explore this and come up with their own methods. Some physics students may want to use formulas that include initial and final velocities, but then you should ask them how they calculate these, and have them explain the formulas they want to use.

As time permits, have students present their solutions, particularly to Part IV and discuss. This should lead nicely to a discussion of the need for limits, which you can cover in the next class period. (Not usually time for this in a 50 minute class)

Remind them of the writing assignment. Alternately, you can have them post a blog to a class website.

Options for the following class:

*Lead them through a discussion of instantaneous velocity using this as a framework. Discuss student's answers to possible or impossible and build off of those. Have a student with each answer (possible and impossible) share.

*As you discuss limits, tie in to the smaller intervals they calculated to answer Part IV.

*Graph keychain position versus time with data you created and passed out or record group data to discuss graphical representations of average and instantaneous velocity. (See follow up activity #1)

Keychain Zipline

Lisa Driskell, Colorado Mesa University
Audrey Malagon, Virginia Wesleyan College

Student Handout

Part I. Construction.

Use the materials in your kit to construct a keychain zipline out of ribbon. Run a few test runs and then secure the ends with tape or by having a teammate hold in a fixed location. The keychain should glide quickly and easily down the zipline, but not too fast! (You'll be timing intervals later).

Part II. Average Velocity

Record the length of your zipline. Be sure to use appropriate units!

Length: ______________

Time how long it takes the keychain to travel the entire zipline. Use seconds (and fractions of seconds) for your units.

Time: ________________

The **average velocity** of the keychain during this trip can be found using the formula

$$\frac{distance\ traveled}{time\ to\ travel}$$

Find the average velocity. What are the units?

Average Velocity: ___________________

Part III. Which end is faster?

Find the point on the ribbon that is 1/3 of the distance from the bottom of the zipline. Mark this with your marker on the ribbon.

You now will find the average velocity for the keychain on the top 2/3 of its trip and on the bottom 1/3. (You may need to run a few trials until you obtain consistent times. You can average your times to use to compute average velocity.)What will you measure in order to do this? Record and explain your calculations below.

Length: _______________ Time trials: _________________________________

Average Velocity for top 2/3: ________________

Average Velocity for bottom 1/3: _______________

Were these the same? How do they compare to each other and to the average velocity of the total trip you found in Part II?

Part IV. *Instantaneous Velocity*

We have been able to measure average velocities by calculating a distance traveled and the time it took to travel that distance. Is it possible to know exactly how fast the keychain is traveling when it passes the 2/3 mark? What about the halfway mark? What about how fast it crashes into the floor/table/wall at the end of the zipline? Discuss this with your teammates and record your thoughts below.

Mark the points that are 1/4 of the distance from the top and 1/4 of the distance from the bottom of the zipline.

Find the average velocity of the zipliner between these two new marks (middle 1/2).

Length: __________. Time trials: __________________.
Average velocity: _____________.

Mark the points that are approximately 1/3 of the distance from the top and approximately 1/3 of the distance from the bottom of the zipline.

Find the average velocity of the zipliner between these two new marks (middle 1/3).

Length: __________. Time trials: __________________.
Average velocity: _____________.

(Time permitting) Choose another interval. Mark a point above the 1/2 mark and below the 1/2 mark and find the average velocity of the zipliner on your new interval

Length: __________. Time trials: __________________.

Average velocity: _____________.

Part IV. Revisiting Instantaneous Velocity

Mark the halfway point on your zipline.

We have been able to measure average velocities by calculating distance traveled and the time it took to travel that distance. Is it possible to know exactly how fast the keychain is traveling the moment it passes the halfway mark? If so, find this instantaneous velocity. If not, explain why not.

Writing Assignment: Prepare a blog entry discussing today's activity. Summarize your group's findings and discuss the question in Part IV. Please type, print and bring to the next class.

Guidelines for Assessing the Activity and the Writing Assignment:

This is designed to be a formative assessment and much of the initial assessment during the activity is done in the form of "checking in" with students either in their small groups or as a large group to ensure that everyone is on the right track. Some things to especially watch out for and correct as needed during the activity

- Ensure that each group is correctly calculating average velocities and are using appropriate units. Review with individuals or as a large group if needed.
- Check that students have correctly marked the 1/3 and 2/3 marks on their zipline and that they understand they are measuring from the top (starting point) of the zipline. Correct as needed.
- Once students have computed the average velocities on different intervals, ask them why the average velocities are different. It will be eye opening to many students that the velocity changes as the zipliner travels. This is crucial to their later understanding of instantaneous velocity or velocity "at a point" and to the idea of velocity as a function of time.

In the writing assignment

- Look for any connections between average velocities and instantaneous velocity, though this may not be there yet.
- Look for an argument that instantaneous rate of change cannot be calculated because two data points are needed. This shows that the student understands the need for more mathematical tools beyond average velocity calculations.
- Look for any discussion that smaller intervals give better approximations to instantaneous velocity.

Based on the information you receive from the students writing assignments and discussions during the activity, you can tailor follow up activities to meet the students' needs. I typically grade this writing assignment only on effort, using it as tool to form future lessons. One suggestion for the next day is provided below.

Follow Up Activity #1

Keychain Zipline Data

Instructor's Guide

Materials Needed: Student Handout, Rulers or Straightedge

Learning Objectives:

- Graphically represent average and instantaneous velocity as slopes of lines
- Interpret average rate of change and instantaneous velocity in context other than velocity

Context:

This activity is usually done the day after the zipline activity. Instructor uses a copy of the student handout on a document camera, guiding the students through filling out the handout as a class. The second part could be done in small groups, with the instructor circulating to help.

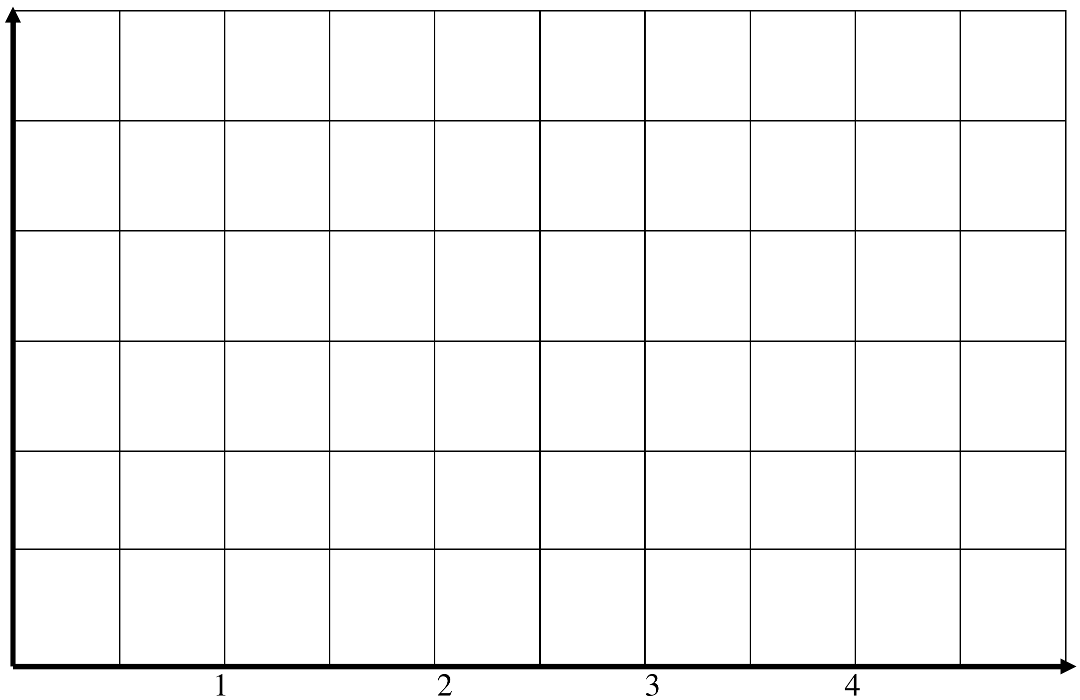
The Activity:

Ask students what the position of the zipliner is when time is 0 and then fill in 0 for that data point. Discuss why the position function is increasing and make sure students understand that we are measuring how far the zipliner is from the start, so these numbers should be growing. (You can comment that this is fictional data from a much longer zipline or you could replace with actual student data if you have that available.)

t = Time (sec)	0	1	2	3	4
s = Position from end (cm)		50	150	275	425

Discuss and label the axes below as time (seconds) and position (cm). Then have students plot each point on their individual graph. Discuss the shape of the graph (increasing) and reinforce that this is NOT the shape of the zipline (which goes down). Again discuss why the numbers for position get larger and how we see that on the graph.

Plot Position as a function of time. Label the axes!



Ask students to find the average velocity between t=2 and t=4. Discuss with them what they are calculating. How do we "see" the change in position graphically (as a vertical distance between two position values). How do we "see" the change in time (as a horizontal distance between two t-values). When we divide these quantities, what are we finding? (slope of the line between the points with t=2 and t=4).

Repeat with the next question: Find the average velocity between t=2 and t=3. Draw the representation on the graph.

Ask students "How do we represent the instantaneous velocity at t=2 on the graph?" Discuss the idea that we need two points to form a slope and relate this back to the concepts discussed in the keychain zipline activity. Discuss how we could get better and better approximations by drawing secant lines between t=2 and t=2.5 and then t=2 and 2.01. Sometimes I will challenge a student o give me a "great approximation" (t=2 and t=2.0001) and then ask another student to "beat" his or her approximation with a better one (t=2 and t=2.00000001). You might also ask if t=1.9999 and t=2 is also a good approximation. Guide students to realize the slope of the tangent line at t=2 represents the instantaneous velocity since it is what each of these great approximations are approaching.

For this problem, it may be helpful to do (a) and (b) as a large group or have small groups report on (a) and (b) before moving on. Encourage students to use the units as a guide to interpreting slope.

Recent pollution has affected the growth of a certain species of fish in a local stream. A team of researchers samples the fish population at various times throughout the year and measures the number of fish observed at a fixed observation point. Their data is recorded in the table below.

Month (t=1: Jan)	1	5	6	7	8
Number of fish observed at observation point	20	50	40	35	32

a) What does the slope of the secant line between t=1 and t=5 tell you in this problem? In general, what does the slope of a secant line tell you?

(Average rate of change of fish population between January and May in fish per month. Average rate of change of fish population in fish per month.)

b) What does the slope of a tangent line tell you?

(Instantaneous rate of change of fish population in fish per month)
Students may be uncomfortable with "per month" as a unit for instantaneous rate of change. Discuss.

c) Find the slope of the secant lines below.

Between t=1 and t=5	
Between t=1 and t=5	
Between t=5 and t=7	
Between t=6and t=7	
Between t=7 and t=8	

d) What is a good guess for the slope of the tangent line at t=7?

Students should observe that the best approximations will be the secant lines between t=6 and t=7 and between t=7 and t=8, and should make a guess between those values. Take care when using the word "average" here if you intend to average the values and stress that we are looking for an approximation of the instantaneous rate of change.

e) How fast is the fish population declining in July?

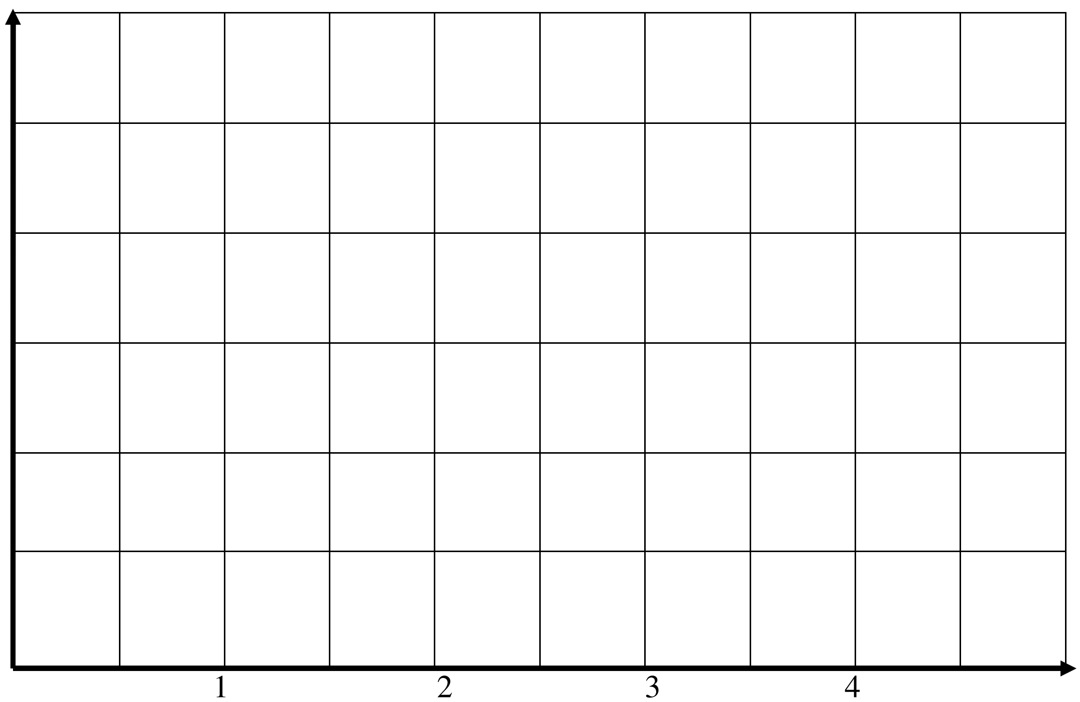
Students should recognize this as the same question being asked in (d).

Keychain Zipline Data

Student Handout

t = Time (sec)	0	1	2	3	4
s = Position from end (cm)		50	150	275	425

Plot Position as a function of time. Label the axes!



Find the average velocity between t=2 and t=4. Draw the representation on the graph.

Find the average velocity between t=2 and t=3. Draw the representation on the graph.

How do we represent the instantaneous velocity at t=2 on the graph?

Recent pollution has affected the growth of a certain species of fish in a local stream. A team of researchers samples the fish population at various times throughout the year and measures the number of fish observed at a fixed observation point. Their data is recorded in the table below.

Month (t=1: Jan)	1	5	6	7	8
Number of fish observed at observation point	20	50	40	35	32

a) What does the slope of the secant line between t=1 and t=5 tell you in this problem? In general, what does the slope of a secant line tell you?

b) What does the slope of a tangent line tell you?

c) Find the slope of the secant lines below.

Between t=1 and t=5	
Between t=1 and t=5	
Between t=5 and t=7	
Between t=6and t=7	
Between t=7 and t=8	

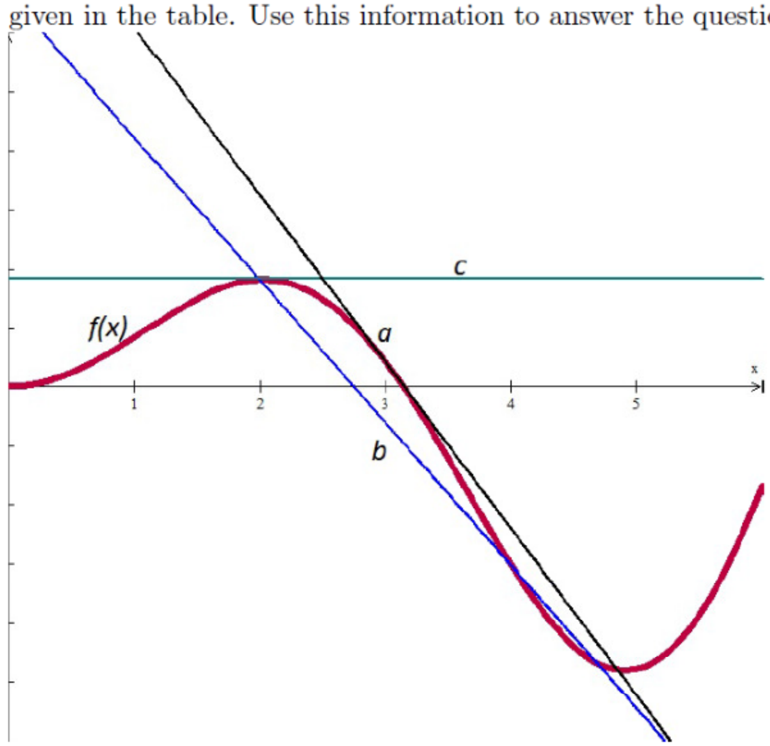
d) What is a good guess for the slope of the tangent line at t=7?

e) How fast is the fish population declining in July?

Final Exam Question to Assess Student Learning Outcomes:

After continuing to develop these ideas throughout the calculus course, I give the following final exam question to assess student learning outcomes.

8. The function $f(x)$ is graphed below along with lines a, b and c. The slopes of these lines are given in the table. Use this information to answer the questions below.



Line	Slope of Line
a	-2.83
b	-2.422
c	0

(a) (3 points) Find $f'(3)$.

(b) (3 points) Find the average rate of change of $f(x)$ from $x = 2$ to $x = 4$.